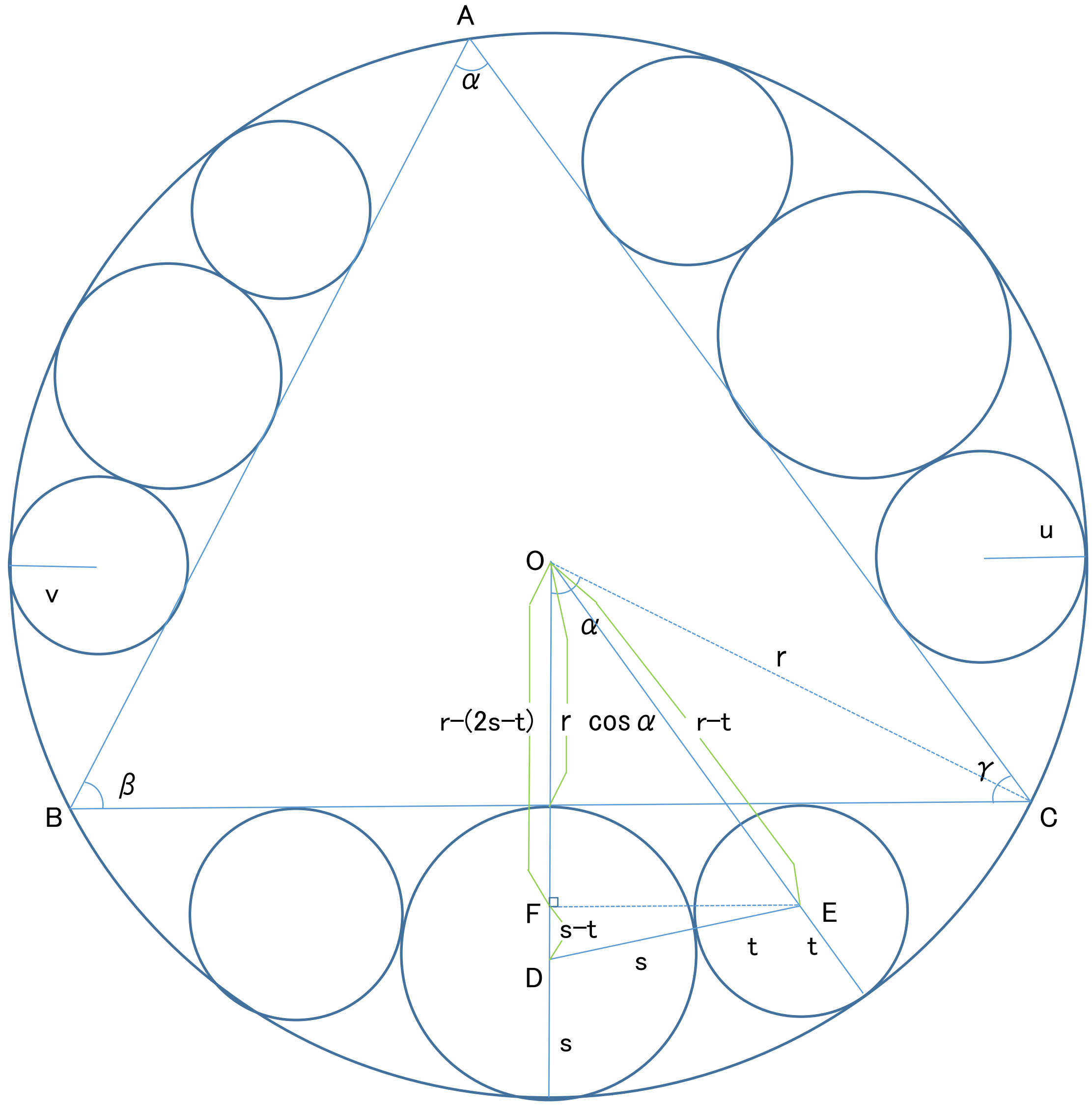
令和５年５月の問題ーNo.2



外円、甲円、乙円、丁円、己円の半径をそれぞれ r, s, t, u, v とし、
また、△ABCの３つの角を α、β、γ とする。

△DEFより、線分EF = $\sqrt{(s+t)^2-(s-t)^2}=2\sqrt{st}$

△OEFより、 $[r-(2s-t)]^2+(2\sqrt{st})^2=(r-t)^2$ $r(s-t)=s^2$ ———①

$r-2s=r\cdot\cos(\alpha)$ $s=\dfrac{r(1-\cos(\alpha))}{2}$ を、式①に代入すると、

$r\left(\dfrac{r(1-\cos(\alpha))}{2}-t\right)=\dfrac{r^2(1-\cos(\alpha))^2}{4}$ $r=\dfrac{4t}{1-(\cos(\alpha))^2}$ ———②

$r=\dfrac{4t}{(\sin(\alpha))^2}$ 丁円、己円についても同様に解くと、 $r=\dfrac{4u}{(\sin(\beta))^2}$ $r=\dfrac{4v}{(\sin(\gamma))^2}$

$\dfrac{4t}{(\sin(\alpha))^2}=\dfrac{4u}{(\sin(\beta))^2}=\dfrac{4v}{(\sin(\gamma))^2}$ $\sin(\beta)=\sqrt{\dfrac{u}{t}}\cdot\sin(\alpha)$ $\sin(\gamma)=\sqrt{\dfrac{v}{t}}\cdot\sin(\alpha)$

$\sin(\beta+\gamma)=\sin(\beta)\cdot\cos(\gamma)+\cos(\beta)\cdot\sin(\gamma)=\sin(\pi-\alpha)=\sin(\alpha)$

$\sin(\beta+\gamma)=\sqrt{\dfrac{u}{t}}\cdot\sin(\alpha)\cdot\sqrt{1-\dfrac{v}{t}\cdot(\sin(\alpha))^2}+\sqrt{1-\dfrac{u}{t}\cdot(\sin(\alpha))^2}\cdot\sqrt{\dfrac{v}{t}}\cdot\sin(\alpha)=\dfrac{\sin(\alpha)}{t}\left(\sqrt{u\left(t-v(\sin(\alpha))^2\right)}+\sqrt{v\left(t-u(\sin(\alpha))^2\right)}\right)=\sin(\alpha)$

$\sqrt{u\left(t-v(\sin(\alpha))^2\right)}+\sqrt{v\left(t-u(\sin(\alpha))^2\right)}=t$ $2\sqrt{uv\left(t-v(\sin(\alpha))^2\right)\left(t-u(\sin(\alpha))^2\right)}=t^2-(u+v)t+2uv(\sin(\alpha))^2$

$4uvt^2\left[1-(\sin(\alpha))^2\right]=[t-(u+v)]^2t^2$ $4uv(\cos(\alpha))^2=[t-(u+v)]^2$ $(\cos(\alpha))^2=\dfrac{[t-(u+v)]^2}{4uv}$

式②より、 $2r=\dfrac{8t}{1-(\cos(\alpha))^2}=\dfrac{8t}{1-\dfrac{[t-(u+v)]^2}{4uv}}=\dfrac{32uvt}{4uv-[t-(u+v)]^2}$

$t=\dfrac{3}{2}$ $u=\dfrac{2}{2}$ $v=\dfrac{1}{2}$ を代入すると、 $2r=12$

外円の直径 = 12寸