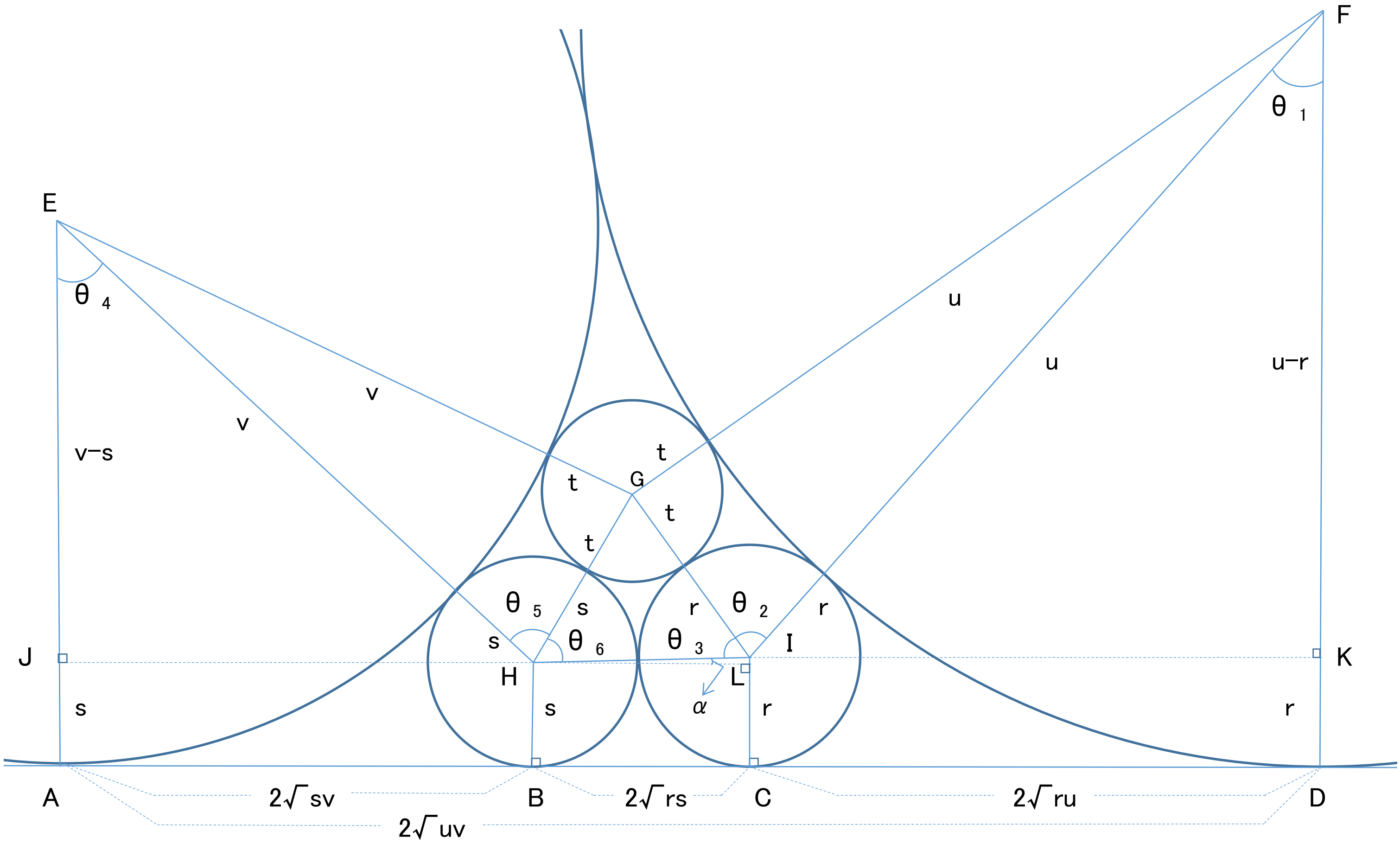

甲、乙、丙、大、小の円の半径をそれぞれ r, s, t, u, v とする。

線分AB = $\sqrt{(v+s)^2-(v-s)^2} = 2\sqrt{sv}$　　同様に、 BC = $2\sqrt{rs}$　CD = $2\sqrt{ru}$　AD = $2\sqrt{uv}$　　$\sqrt{sv}+\sqrt{rs}+\sqrt{ru}=\sqrt{uv}$

$\sqrt{r}\cdot(\sqrt{u}+\sqrt{s}) = \sqrt{v}\cdot(\sqrt{u}-\sqrt{s})$　　———①

△FIKより、　$\cos(\theta_1) = \dfrac{u-r}{u+r}$　　$\sin(\theta_1) = \dfrac{2\sqrt{ru}}{u+r}$

△EHJより、　$\cos(\theta_4) = \dfrac{v-s}{v+s}$　　$\sin(\theta_4) = \dfrac{2\sqrt{sv}}{v+s}$

△HILより、　$\cos(\alpha) = \dfrac{2\sqrt{rs}}{r+s}$　　$\sin(\alpha) = \dfrac{r-s}{r+s}$

△IGFより、　$(u+t)^2 = (r+u)^2+(r+t)^2-2(r+u)(r+t)\cdot\cos(\theta_2)$　　$\cos(\theta_2) = 1-\dfrac{2ut}{(r+u)(r+t)}$　　$\sin(\theta_2) = \dfrac{\sqrt{4rut(r+u+t)}}{(r+u)(r+t)}$

△IGHより、　$(s+t)^2 = (r+s)^2+(r+t)^2-2(r+s)(r+t)\cdot\cos(\theta_3)$　　$\cos(\theta_3) = 1-\dfrac{2st}{(r+s)(r+t)}$　　$\sin(\theta_3) = \dfrac{\sqrt{4str(r+s+t)}}{(r+s)(r+t)}$

$\theta_2+\theta_3-(\alpha+\theta_1) = \dfrac{\pi}{2}$　　$\theta_2+\theta_3 = \dfrac{\pi}{2}+(\alpha+\theta_1)$　　$\cos(\theta_2+\theta_3) = \cos\left[\dfrac{\pi}{2}+(\alpha+\theta_1)\right] = -\sin(\alpha+\theta_1)$

$$\cos(\theta_2+\theta_3) = \cos(\theta_2)\cdot\cos(\theta_3)-\sin(\theta_2)\cdot\sin(\theta_3) = \left[1-\frac{2ut}{(r+u)\cdot(r+t)}\right]\cdot\left[1-\frac{2st}{(r+s)\cdot(r+t)}\right]-\frac{4tr\cdot\sqrt{su(r+u+t)\cdot(r+s+t)}}{(r+t)^2\cdot(r+u)\cdot(r+s)}$$

$$\sin(\alpha+\theta_1) = \sin(\alpha)\cdot\cos(\theta_1)+\cos(\alpha)\cdot\sin(\theta_1) = \frac{r-s}{r+s}\cdot\frac{u-r}{u+r}+\frac{2\sqrt{rs}}{r+s}\cdot\frac{2\sqrt{ru}}{u+r} = \frac{(r-s)\cdot(u-r)+4r\cdot\sqrt{su}}{(r+s)\cdot(u+r)}$$

$$1-\frac{2st}{(r+s)\cdot(r+t)}-\frac{2ut}{(r+u)\cdot(r+t)}+\frac{4t^2su-4tr\cdot\sqrt{su(r+u+t)\cdot(r+s+t)}}{(r+t)^2\cdot(r+u)\cdot(r+s)}+\frac{(r-s)\cdot(u-r)+4r\cdot\sqrt{su}}{(r+s)(r+u)} = 0$$

$$(r+t)^2(r+s)(r+u)-2st(r+t)(r+u)-2ut(r+t)(r+s)+4t^2su-4tr\cdot\sqrt{su(r+u+t)(r+s+t)}+(r+t)^2\left[(r-s)(u-r)+4r\sqrt{su}\right] = 0$$

$$(r+t)^2\cdot\left[(r+s)(r+u)+(r-s)(u-r)+4r\sqrt{su}\right] = 2t\cdot\left[s(r+t)(r+u)+u(r+t)(r+s)-2tsu+2r\cdot\sqrt{su(r+u+t)(r+s+t)}\right]$$

$$2r(r+t)^2(u+s+2\sqrt{su}) = 2t\left[rs(r+u+t)+ru(r+s+t)+2r\cdot\sqrt{su(r+u+t)(r+s+t)}\right]$$

$$(r+t)^2(\sqrt{u}+\sqrt{s})^2 = t(\sqrt{s(r+u+t)}+\sqrt{u(r+s+t)})^2$$

$$(r+t)(\sqrt{u}+\sqrt{s}) = \sqrt{t}(\sqrt{s(r+u+t)}+\sqrt{u(r+s+t)})$$

$$[(r+t)-\sqrt{t(r+s+t)}]\cdot\sqrt{u}+(r+t)\cdot\sqrt{s} = \sqrt{st(u+r+t)}$$

$$[(r+t)-\sqrt{t(r+s+t)}]^2\cdot u+2\cdot(r+t)\cdot\sqrt{s}\cdot[(r+t)-\sqrt{t(r+s+t)}]\cdot\sqrt{u}+(r+t)^2s = st(u+r+t)$$

$\left[2\cdot\sqrt{t(r+s+t)}-\left(r+2t\right)\right]u-2\cdot\sqrt{s}\left[(r+t)-\sqrt{t(r+s+t)}\right]\cdot\sqrt{u}-rs=0$

(i)　$\sqrt{u}=\dfrac{\sqrt{s}\cdot\left[(r+t)-\sqrt{t(r+s+t)}\right]+\sqrt{st}\cdot(\sqrt{r+s+t}-\sqrt{t})}{2\cdot\sqrt{t(r+s+t)}-\left(r+2t\right)}=\dfrac{r\sqrt{s}}{2\cdot\sqrt{t(r+s+t)}-\left(r+2t\right)}$

(ii)　$\sqrt{u}=\dfrac{\sqrt{s}\cdot\left[(r+t)-\sqrt{t(r+s+t)}\right]-\sqrt{st}\cdot(\sqrt{r+s+t}-\sqrt{t})}{2\cdot\sqrt{t(r+s+t)}-\left(r+2t\right)}=-\sqrt{s}$　<0 なので除外する。

$\sqrt{u}=\dfrac{r\sqrt{s}}{2(\sqrt{t(r+s+t)}-t)-r}$　　―――②

△HGEより、　$(v+t)^2=(s+v)^2+(s+t)^2-2(s+v)(s+t)\cdot\cos\left(\theta_5\right)$　　$\cos\left(\theta_5\right)=1-\dfrac{2vt}{(s+v)(s+t)}$　　$\sin\left(\theta_5\right)=\dfrac{\sqrt{4svt(s+v+t)}}{(s+v)(s+t)}$

△HGIより、　$(r+t)^2=(s+r)^2+(s+t)^2-2(s+r)(s+t)\cdot\cos\left(\theta_6\right)$　　$\cos\left(\theta_6\right)=1-\dfrac{2rt}{(s+r)(s+t)}$　　$\sin\left(\theta_6\right)=\dfrac{\sqrt{4rst(r+s+t)}}{(s+r)(s+t)}$

$\theta_5+\theta_6+\alpha-\theta_4=\dfrac{\pi}{2}$　　$\theta_5+\theta_6=\dfrac{\pi}{2}-\left(\alpha-\theta_4\right)$　　$\cos\left(\theta_5+\theta_6\right)=\cos\left[\dfrac{\pi}{2}-\left(\alpha-\theta_4\right)\right]=\sin\left(\alpha-\theta_4\right)$

$\cos\left(\theta_5+\theta_6\right)=\cos\left(\theta_5\right)\cdot\cos\left(\theta_6\right)-\sin\left(\theta_5\right)\cdot\sin\left(\theta_6\right)=\left[1-\dfrac{2vt}{(s+v)\cdot(s+t)}\right]\cdot\left[1-\dfrac{2rt}{(s+r)\cdot(s+t)}\right]-\dfrac{4st\cdot\sqrt{rv(r+s+t)(s+v+t)}}{(s+t)^2(s+r)(s+v)}$

$\sin\left(\alpha-\theta_4\right)=\sin\left(\alpha\right)\cdot\cos\left(\theta_4\right)-\cos\left(\alpha\right)\cdot\sin\left(\theta_4\right)=\dfrac{r-s}{r+s}\cdot\dfrac{v-s}{v+s}-\dfrac{2\sqrt{rs}}{r+s}\cdot\dfrac{2\sqrt{sv}}{v+s}=\dfrac{(r-s)\cdot(v-s)-4s\cdot\sqrt{rv}}{(r+s)(v+s)}$

$1-\dfrac{2rt}{(s+r)\cdot(s+t)}-\dfrac{2vt}{(s+v)\cdot(s+t)}+\dfrac{4t^2rv-4st\cdot\sqrt{rv(r+s+t)\cdot(s+v+t)}}{(s+t)^2\cdot(s+r)\cdot(s+v)}=\dfrac{(r-s)\cdot(v-s)-4s\cdot\sqrt{rv}}{(r+s)(v+s)}$

$(s+t)^2(s+r)(s+v)-2rt(s+t)(s+v)-2vt(s+t)(s+r)+4t^2rv-4st\cdot\sqrt{rv(r+s+t)(s+v+t)}=(s+t)^2\left[(r-s)(v-s)-4s\cdot\sqrt{rv}\right]$

$(s+t)^2\left[(s+r)(s+v)-(r-s)(v-s)+4s\cdot\sqrt{rv}\right]=2t\left[r(s+t)(s+v)+v(s+t)(s+r)-2trv+2s\cdot\sqrt{rv(r+s+t)(s+v+t)}\right]$

$2s(s+t)^2\left(r+v+2\sqrt{rv}\right)=2t\left[rs(s+v+t)+vs(r+s+t)+2s\cdot\sqrt{rv(r+s+t)(s+v+t)}\right]$

$(s+t)^2(\sqrt{r}+\sqrt{v})^2=t(\sqrt{r(s+v+t)}+\sqrt{v(r+s+t)})^2$

$(s+t)(\sqrt{r}+\sqrt{v})=\sqrt{t}(\sqrt{r(s+v+t)}+\sqrt{v(r+s+t)})$

$[(s+t)-\sqrt{t(r+s+t)}]\cdot\sqrt{v}+(s+t)\cdot\sqrt{r}=\sqrt{rt(v+s+t)}$

$[(s+t)-\sqrt{t(r+s+t)}]^2v+2(s+t)\cdot\sqrt{r}[(s+t)-\sqrt{t(r+s+t)}]\cdot\sqrt{v}+(s+t)^2r=rt(v+s+t)$

$\left[2\cdot\sqrt{t(r+s+t)}-\left(s+2t\right)\right]v-2\cdot\sqrt{r}\left[(s+t)-\sqrt{t(r+s+t)}\right]\cdot\sqrt{v}-rs=0$

(i)　$\sqrt{v}=\dfrac{\sqrt{r}\cdot\left[(s+t)-\sqrt{t(r+s+t)}\right]+\sqrt{rt}\cdot(\sqrt{r+s+t}-\sqrt{t})}{2\cdot\sqrt{t(r+s+t)}-\left(s+2t\right)}=\dfrac{s\sqrt{r}}{2\cdot\sqrt{t(r+s+t)}-\left(s+2t\right)}$

(ii)　$\sqrt{v}=\dfrac{\sqrt{r}\cdot\left[(s+t)-\sqrt{t(r+s+t)}\right]-\sqrt{rt}\cdot(\sqrt{r+s+t}-\sqrt{t})}{2\cdot\sqrt{t(r+s+t)}-\left(s+2t\right)}=-\sqrt{r}$　<0 なので除外する。

$\sqrt{v}=\dfrac{s\sqrt{r}}{2(\sqrt{t(r+s+t)}-t)-s}$　　―――③

$2(\sqrt{t(r+s+t)}-t)=A$　とおくと、式②、③より、　$\sqrt{u}=\dfrac{r\sqrt{s}}{A-r}$　　$\sqrt{v}=\dfrac{s\sqrt{r}}{A-s}$

式①に代入すると　$\sqrt{u}+\sqrt{s}=\dfrac{s}{A-s}(\sqrt{u}-\sqrt{s})$　　$\left(2s-A\right)\cdot\sqrt{u}=A\sqrt{s}$

$\left(2s-A\right)\cdot\dfrac{r\sqrt{s}}{A-r}=A\sqrt{s}$　　$\left(2s-A\right)r=A(A-r)$　　$A^2=2rs$　　$A=\sqrt{2rs}$

$\sqrt{2t(r+s+t)}=\sqrt{2t}+\sqrt{rs}$　　$2t(r+s+t)=2t^2+rs+2t\sqrt{2rs}$　　$2t\left(r+s-\sqrt{2rs}\right)=rs$

$2t=\dfrac{rs}{r+s-\sqrt{2rs}}$

$r=\dfrac{45}{2}$　$s=\dfrac{40}{2}$　を代入すると、　$2t=36$

∴ 両円の直径 ＝ 36寸