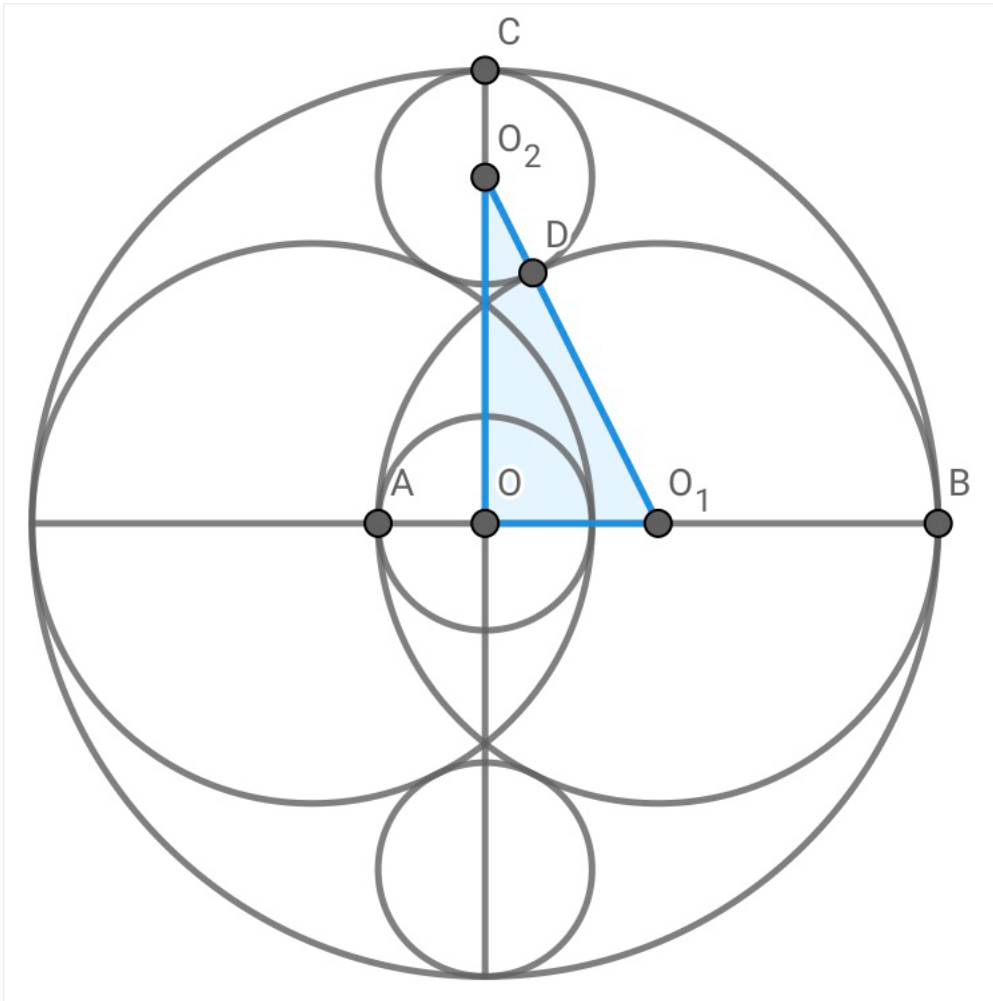


令和4年9月の問題 - No.2



大球・甲球2個の中心，甲球に外接する乙球2個の中心を通る断面を図示する．記号を以下とする．

$$(\text{大球 } O \text{ 半径}) = r \quad (r > 0)$$

$$(\text{甲球 } O_1 \text{ 半径}) = ar \quad (0 < a < 1) \text{ とすると}$$

$$(\text{乙球 } O_2 \text{ 半径}) = OA = AB - OB = (2a - 1)r$$

$\triangle OO_1O_2$ (青色) に三平方の定理を用いる．

$$OO_1 = O_1A - OA = ar - (2a - 1)r = (1 - a)r$$

$$OO_2 = OC - O_2C = r - (2a - 1)r = 2(1 - a)r$$

$$O_1O_2 = O_1D + O_2D = ar + (2a - 1)r = (3a - 1)r$$

$$(OO_1)^2 + (OO_2)^2 = (O_1O_2)^2 \text{ から}$$

$$\{(1-a)r\}^2 + \{2(1-a)r\}^2 = \{(3a-1)r\}^2$$

$$a^2 + a - 1 = 0 \quad (r \neq 0)$$

$$\therefore a = \frac{\sqrt{5}-1}{2} = 0.6180 \dots \quad (0 < a < 1)$$

$$(\text{甲球径}) = 2ar = (\sqrt{5}-1)r = 987 \text{ 寸 より}$$

$$(\text{大球径}) = 2r = \frac{987(\sqrt{5}+1)}{2} = 1596.9995 \dots \text{ 寸}$$

$$(\text{乙球径}) = 2(2a-1)r = \frac{987(3-\sqrt{5})}{2} = 377.0004 \dots \text{ 寸}$$

●答え 乙球径：約377寸 (大球径：約1597寸)