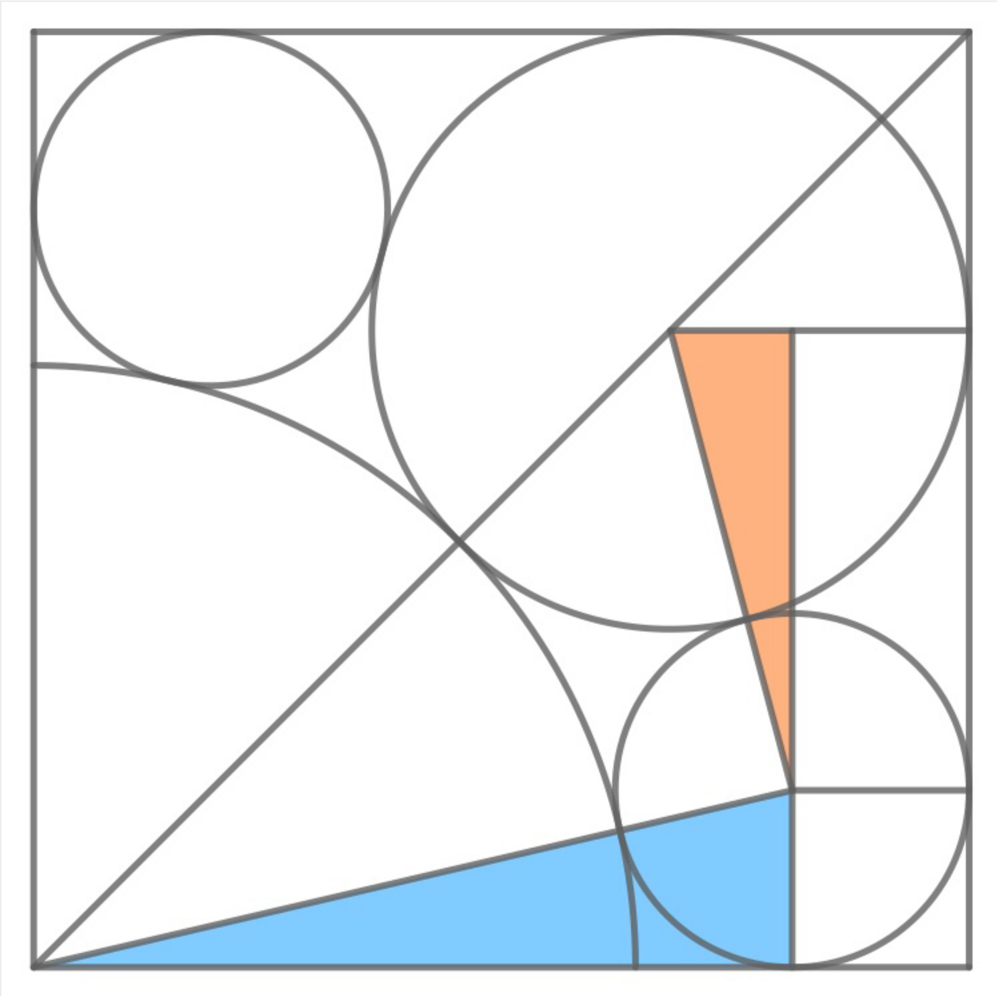
令和４年７月の問題 - No.1



原図は上下左右対称のため, 右上 $\frac{1}{4}$ のみを図示する
(対角線, 橙色と青色の直角三角形を追記)

各円の半径を以下とする (乙円 < 甲円 < 丙円)

(甲円の半径) $= r \qquad (r > 0)$

(乙円の半径) $= ar \qquad (0 < a < 1)$

(丙円の半径) $= br \qquad (b > 1)$

原図にて (正方形の $\frac{1}{2}$ 辺) $= c$ とすると
図示より (対角線) $= (1 + b + \sqrt{2})r$ から

$$c = \frac{1}{\sqrt{2}}(1 + b + \sqrt{2})r$$

橙色の直角三角形に三平方の定理を用いると

$$\{\frac{1}{\sqrt{2}}(1+b)-a\}^2r^2+(1-a)^2r^2=(1+a)^2r^2$$

$$\{\frac{1}{\sqrt{2}}(1+b)-a\}^2=4a \quad (r>0)$$

$$\frac{1}{\sqrt{2}}(1+b)-a=\pm 2\sqrt{a}$$

$(\text{左辺})=\frac{1}{\sqrt{2}}(1+b)-a>\sqrt{2}-1>0$ から

$$b=\sqrt{2}a+2\sqrt{2}\sqrt{a}-1$$

記述を簡素化するため，$\sqrt{a}=x$ とし

$$b=\sqrt{2}x^2+2\sqrt{2}x-1 \quad (0<x<1)$$

水色の直角三角形に三平方の定理を用いると

$$\{\frac{1}{\sqrt{2}}(1+b+\sqrt{2})-a\}^2r^2+a^2r^2 \;=\; (a+b)^2r^2$$

$\sqrt{a}=x,\ \ b=\sqrt{2}x^2+2\sqrt{2}x-1$ を代入し

$$2(1+\sqrt{2})x^4+4\sqrt{2}(1+\sqrt{2})x^3+2(1-\sqrt{2})x^2$$
$$-4(1+\sqrt{2})x=0$$

因数分解し解くと

$$2(1+\sqrt{2})\ x\ (x+1)\ \{x^2-(1-2\sqrt{2})x-2\}=0$$

$$\therefore\ x=\frac{(1-2\sqrt{2})+\sqrt{17-4\sqrt{2}}}{2} \quad (0<x<1)$$

$$(\text{乙円の半径}) = ar = x^2 r$$

$$= \frac{(13 - 4\sqrt{2}) + (1 - 2\sqrt{2})\sqrt{17 - 4\sqrt{2}}}{2}\ r$$

$$= (0.5925\cdots)\ r$$

$$\fallingdotseq\ 0.6\ r$$

$$(\text{丙円の半径}) = br = (\sqrt{2}x^2 + 2\sqrt{2}x - 1)r$$

$$= \frac{(-18 + 15\sqrt{2}) - (4 - 3\sqrt{2})\sqrt{17 - 4\sqrt{2}}}{2}\ r$$

$$= (2.0152\cdots)\ r$$

$$\fallingdotseq\ 2.0\ r$$

$$(\text{正方形の}\ \tfrac{1}{2}\ \text{辺}) = c = \frac{1}{\sqrt{2}}(1 + b + \sqrt{2})r$$

$$= \frac{(17 - 8\sqrt{2}) + (3 - 2\sqrt{2})\sqrt{17 - 4\sqrt{2}}}{2}\ r$$

$$= (3.1320\cdots)\ r$$

$$\fallingdotseq\ 3.1\ r$$

$$(\text{正方形の}\,1\,\text{辺}) = 2c$$

$$= \{(17 - 8\sqrt{2}) + (3 - 2\sqrt{2})\sqrt{17 - 4\sqrt{2}}\}\ r$$

$$= (6.2641\cdots)\ r$$

$$\fallingdotseq\ 6.3\ r$$

答え

(甲円径) ＝ 1 寸 のとき

$$(\text{乙円径}) = \frac{(13-4\sqrt{2})+(1-2\sqrt{2})\sqrt{17-4\sqrt{2}}}{2} \fallingdotseq 0.6 \text{ 寸}$$

$$(\text{丙円径}) = \frac{(-18+15\sqrt{2})-(4-3\sqrt{2})\sqrt{17-4\sqrt{2}}}{2} \fallingdotseq 2.0 \text{ 寸}$$

$$(\text{正方形}) = \{(17-8\sqrt{2})+(3-2\sqrt{2})\sqrt{17-4\sqrt{2}}\} \fallingdotseq 6.3 \text{ 寸}$$